

CAT Averages Formula Sheet

Every formula you need for CAT Quant: basics, weighted average, replacement, speed and alligation

1. Core Definitions

Concept	Formula / Rule
Average	Sum of observations / Number of observations
Sum	Average x Number of observations (always think in totals)
Effect of multiplying all terms by k	Average is multiplied by k
Effect of adding c to all terms	Average increases by c
Average lies between	The smallest and largest observation

2. Averages of Standard Series

Series	Average
First n natural numbers	$(n + 1) / 2$
First n odd numbers	n
First n even numbers	$n + 1$
First n squares	$(n + 1)(2n + 1) / 6$
First n cubes	$n(n + 1)^2 / 4$
Any AP / equally spaced terms	(First term + Last term) / 2 = Middle term (odd count)
n multiples of k (k, 2k, ..., nk)	$k(n + 1) / 2$

3. Weighted and Combined Average

Concept	Formula
Weighted average	$(w_1 \times a_1 + w_2 \times a_2 + \dots) / (w_1 + w_2 + \dots)$
Two groups combined	$(n_1 \times A_1 + n_2 \times A_2) / (n_1 + n_2)$
Ratio of group sizes (alligation)	$n_1 : n_2 = (A_2 - \text{Avg}) : (\text{Avg} - A_1)$, where A_1 is lower and A_2 is higher
Never do this	Add the group averages and divide by 2 unless group sizes are equal

4. Adding, Removing and Replacing Items

Situation	Shortcut
New item added to n items, average changes by d	New item = Old average + $(n + 1) \times d$
Item removed from n items, average changes by d	Removed item = Old average - $(n - 1) \times d$
One item replaced in a group of n	New item = Old item + $n \times (\text{change in average})$

Situation	Shortcut
Batsman, N-th innings raises average by d	New average = $\text{Score} - (N - 1) \times d$
Same group after t years	Average age increases by t (if no one joins or leaves)
Member leaves and new member joins	Compare totals, not just averages

5. Average Speed

Situation	Formula
Equal distances at speeds x and y	$2xy / (x + y)$ (harmonic mean)
Equal distances at x, y, z	$3xyz / (xy + yz + zx)$
Equal times at speeds x and y	$(x + y) / 2$ (simple mean)
General case	Total distance / Total time
Return speed y for round-trip average A, outward speed x	$y = Ax / (2x - A)$

6. Min / Max Logic

Goal	Method
Maximise one value (distinct positive integers)	Set all other values to the smallest allowed (1, 2, 3, ...)
Minimise the largest value	Make all values as equal as possible, then adjust to meet the total
Average of consecutive integers	Equals the middle integer (or mean of the two middle integers)
Sum of 1 to n	$n(n + 1) / 2$

7. Common Mistakes to Avoid

Mistake	Correct approach
Averaging two averages directly	Use weighted average with group sizes
Using the simple mean for speeds over equal distances	Use $2xy / (x + y)$
Forgetting the new item in n + 1	Recount the number of items after every addition
Double-counting a shared term	Use inclusion logic: sum of group totals - overall total
Ignoring integer or distinct conditions	Check feasibility with an example before answering

8. Quick Worked Examples

Example 1: 30 students average 60 and 20 students average 75. Combined average = $(1800 + 1500) / 50 = 66$.

Example 2: A person drives equal distances at 60 and 40 km/h. Average speed = $2 \times 60 \times 40 / 100 = 48$ km/h.

Example 3: Average of 8 people rises by 2.5 when a 65 kg person is replaced. New person = $65 + 8 \times 2.5 = 85$ kg.

Revision tip: Cover the right-hand column, say each formula aloud, then check. Practise with the companion PDF of 35 CAT Averages questions.